Mechanisms and behaviors in complex systems with group interactions

Thomas Robiglio

IT:U Interdisciplinary Transformation University Austria
DNDS, Central European University
Inverse Complexity Lab

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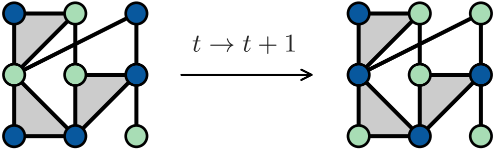
Some references

- Battiston, F. *et al.* The physics of higher-order interactions in complex systems. *Nature Physics* **17,** 1093–1098 (2021)
- Rosas, F. E. *et al.* Disentangling high-order mechanisms and high-order behaviours in complex systems. *Nature Physics* **18,** 476–477 (2022)
- **Robiglio, T. *et al.* Synergistic signatures of group mechanisms in higher-order systems. *arXiv preprint arXiv:2401.11588* (2024)**
- Peel, L. *et al.* Statistical inference links data and theory in network science. *Nature Communications* **13,** 6794 (2022)

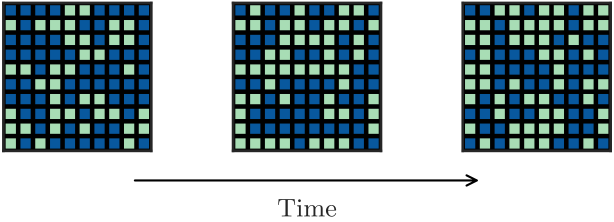
(a)

Mechanism



(b)

Behavior



Mechanisms

Structural and dynamical rules controlling the causal evolution of the system.

Examples:

$$H = -\sum_{i \neq j} J_{ij} s_i s_j, \quad P\left(s_i^t \to s_i^{t+1}\right) = \dots$$

$$\rho_i^{t+1} = \rho_i^t + \mu[1 - \rho_i^t] - \rho_i^t \left\{1 - \prod_j \left[1 - \beta A_{ij}(1 - \rho_j^t)\right]\right\}$$

$$\dot{\theta}_i = \omega_i + \lambda \sum_j A_{ij} \sin(\theta_j - \theta_i)$$

Behaviors

Measurable observables quantifying statistical inter-dependencies between units.

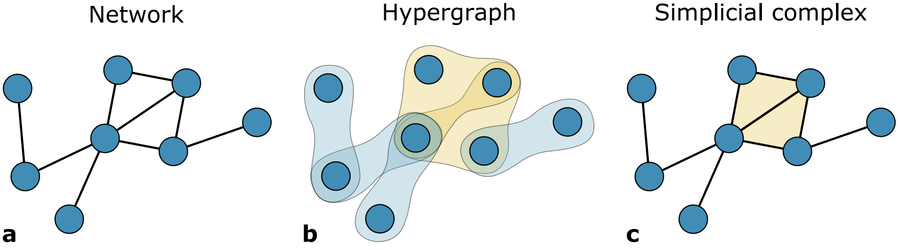
Examples:

- Correlations
- Mutual information
- Autocorrelation
- Transfer entropy
- ...

Beyond pairwise?

Higher-order mechanisms

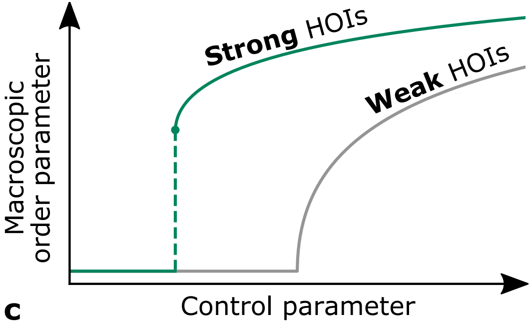
Explicit representations of group interactions[1,2]:



[1] Battiston, F. *et al.* Networks beyond pairwise interactions: Structure and dynamics. *Physics reports* **874,** 1–92 (2020).

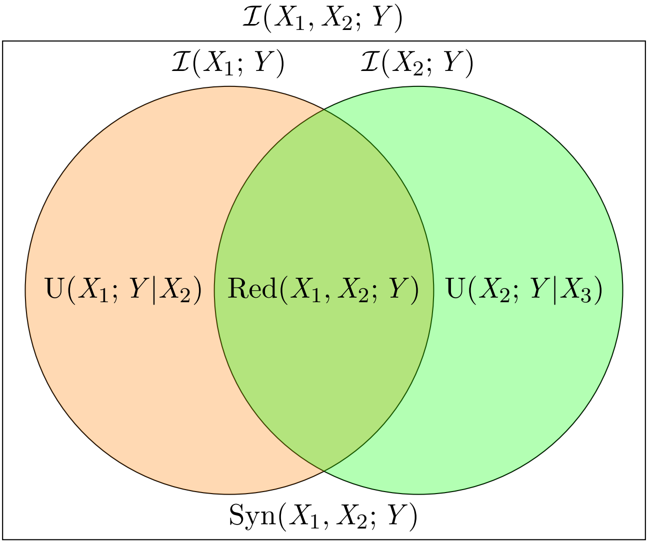
[2] Battiston, F. *et al.* The physics of higher-order interactions in complex systems. *Nature Physics* **17,** 1093–1098 (2021).

Higher-order interactions lead to explosive phenomena:



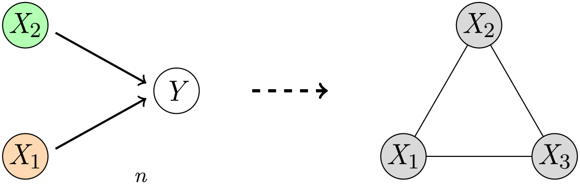
Higher-order behaviors

Information two r.v. X_1 and X_2 contain a third r.v. Y[3]:



[3] Williams, P. L. & Beer, R. D. Nonnegative decomposition of multivariate information. *arXiv preprint arXiv:1004.2515* (2010).

Higher-order behaviors



$$\mathrm{TC}(\mathbf{X}^n) \equiv \sum_{j=1}^{n} \mathcal{H}(X_j) - \mathcal{H}(\mathbf{X}^n) \rightarrow \text{Collective constraints}$$

$$\mathrm{DTC}(\mathbf{X}^n) \equiv \mathcal{H}(\mathbf{X}^n) - \sum_{j=1}^{n} \mathcal{H}(X_j|\mathbf{X}^n_{-j}) \rightarrow \text{Shared randomness}$$

O-information: $\Omega(\mathbf{X}^n) \equiv \mathrm{TC}(\mathbf{X}^n) - \mathrm{DTC}(\mathbf{X}^n)$

Intuitively[4]:

$$\Omega(\mathbf{X}^n) > 0 \rightarrow \text{Redundancy dominates}$$

$$\Omega(\mathbf{X}^n) < 0 \rightarrow \text{Synergy dominates}$$

[4] Rosas, F. E. *et al.* Quantifying high-order interdependencies via multivariate extensions of the mutual information. *Physical Review E* **100,** 032305 (2019).

LOW-ORDER MECHANISM

LOW-ORDER BEHAVIOR

Correlations, MI, Granger caus., *etc.*

HIGHER-ORDER MECHANISM

HIGHER-ORDER BEHAVIOR

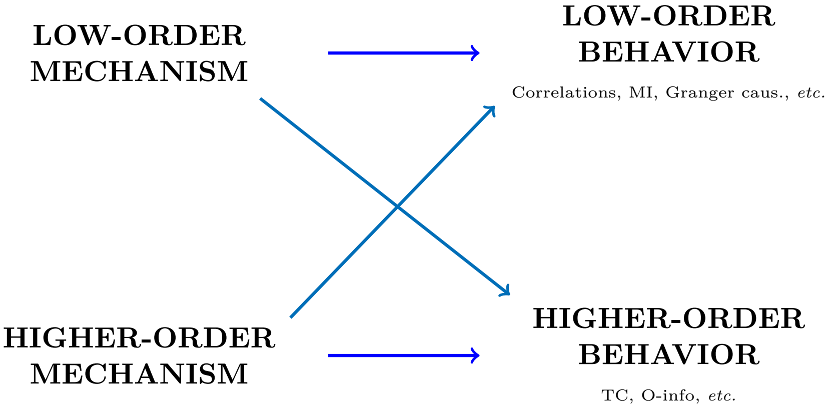
TC, O-info, *etc.*

LOW-ORDER MECHANISM $\longrightarrow$ **LOW-ORDER BEHAVIOR**

Correlations, MI, Granger caus., *etc.*

HIGHER-ORDER MECHANISM $\longrightarrow$ **HIGHER-ORDER BEHAVIOR**

TC, O-info, *etc.*


LOW-ORDER MECHANISM
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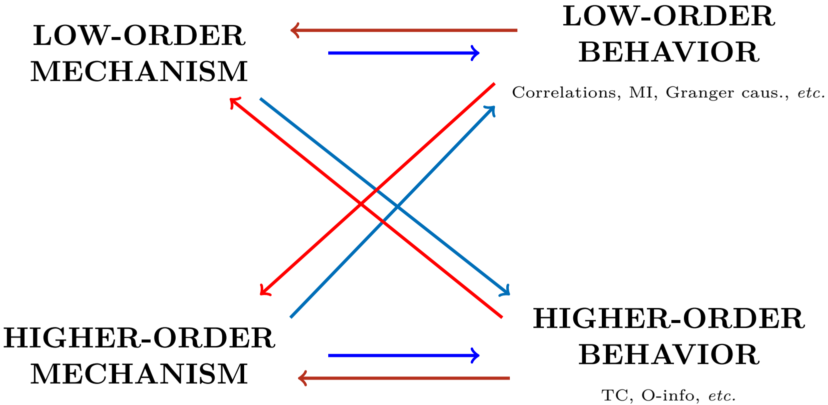
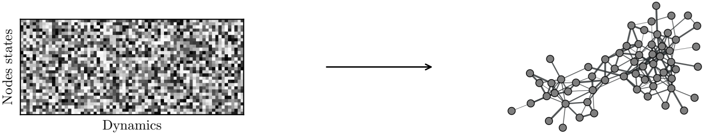

LOW-ORDER MECHANISM
LOW-ORDER BEHAVIOR
Correlations, MI, Granger caus., *etc.*
HIGHER-ORDER MECHANISM
HIGHER-ORDER BEHAVIOR
TC, O-info, *etc.*

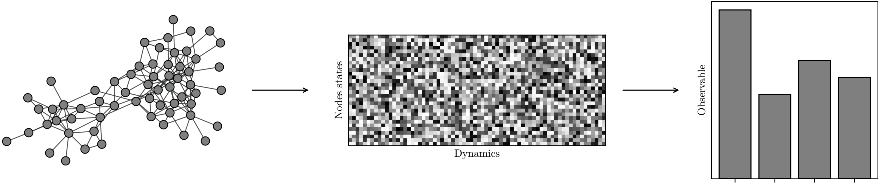
What to do?

1. **Clarify definitions**
2. **Systematic exploration**
3. Full mapping?

The hard task

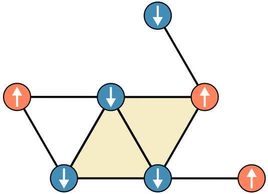


The "easy" task



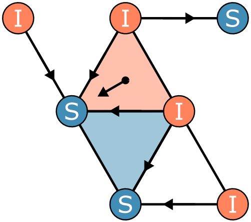
The "easy" task - models

(i) Simplicial Ising model[5]



$$H = -J_0 \sum_{i=1}^{N} S^i - \sum_{\ell=1}^{\ell_{\max}} J_\ell \sum_{\{\sigma \in \mathcal{K} : |\sigma| = \ell\}} \left[2 \bigotimes_{i \in \sigma} S^i - 1 \right]$$

(ii) Simplicial contagion model[6]



Pairwise infection rate: β_1
Group infection rate: β_2
Recovery rate: μ

[5] Robiglio, T. *et al.* Higher-order Ising model on hypergraphs. *arXiv preprint arXiv:2411.19618* (2024).

[6] Iacopini, I. *et al.* Simplicial models of social contagion. *Nature communications* **10,** 2485 (2019).

The "easy" task - behavior observable

Generalize O-information to time-lagged correlations[7]:

1. Considering n variables $\mathbf{X} = (X_1, ..., X_n)$ on which we compute the O-information Ω_n,
2. Add a new variable Y,
3. Compute the variation of O-information:

$$\Delta_n = \Omega_{n+1}(\mathbf{X}, Y) - \Omega_n(\mathbf{X})$$

4. Condition of the past of Y:

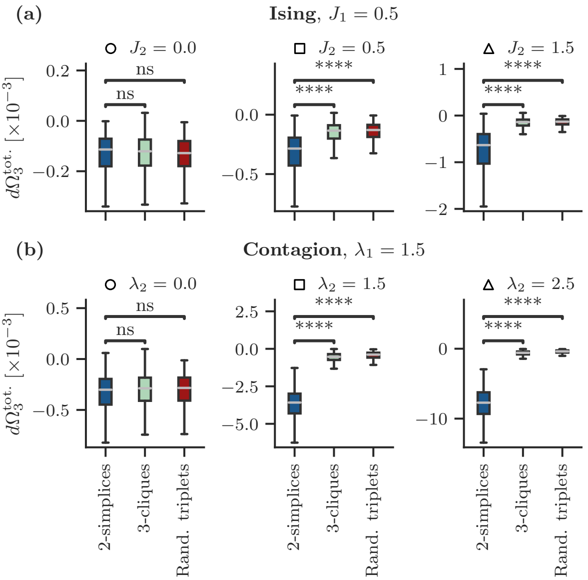
$$d\Omega_n(Y; \mathbf{X}) \equiv (1-n)\mathcal{I}(Y; \mathbf{X}| Y_0) + \sum_{j=1}^{n} \mathcal{I}(Y; \mathbf{X}_{-j}| Y_0)$$

We define **total** dynamical O-information:

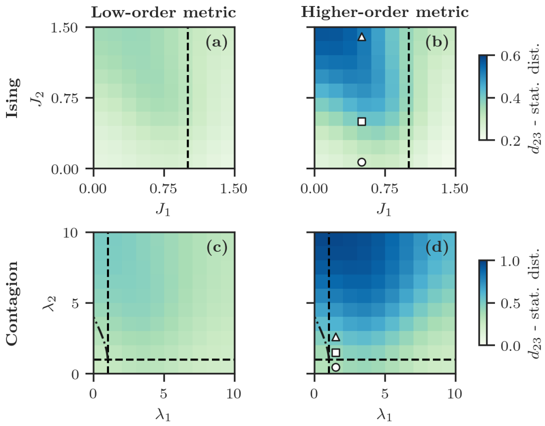
$$d\Omega_n^{\text{tot.}}(\mathbf{X}) \equiv \sum_{j=1}^{n} d\Omega_{n-1}(X_j; \mathbf{X}_{-j})$$

[7] Stramaglia, S. *et al.* Quantifying dynamical high-order interdependencies from the o-information: an application to neural spiking dynamics. *Frontiers in Physiology* **11**, 595736 (2021).

Synergistic signatures



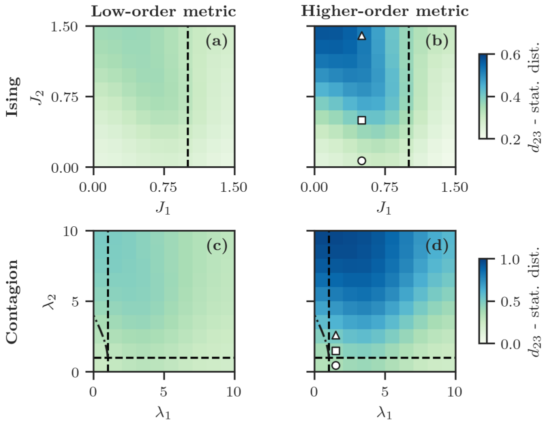
Insufficiency of low-order metrics



Transfer entropy: $\mathcal{T}(X;Y) \equiv \mathcal{I}(X_t, Y_{t+1} | Y_t)$

Statistical distance: $d(P_2, P_3) = \frac{1}{2}\sum_{x\in\chi} |P_2(x) - P_3(x)|$

Landscape

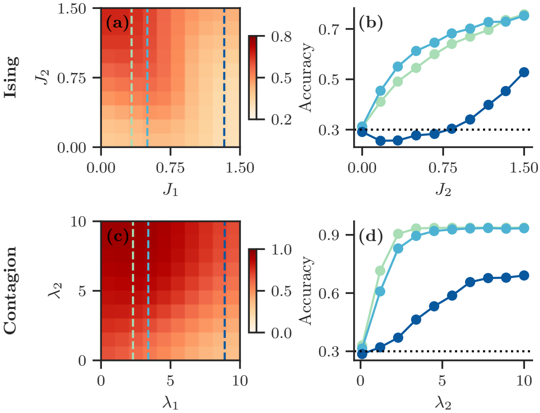


Pairwise Ising: $J_1^{\text{cr.}} = T/\langle k_1 \rangle$

Contagion: pairwise $\lambda_1^{\text{cr.}} = 1$, disc. transition $\lambda_1 = 2\sqrt{\lambda_2} - \lambda_2$

Downstream tasks

Given: pairwise structure[8] and $\langle k_2 \rangle$
Task: find the true higher-order interactions



[8] Young, J.-G. *et al.* Hypergraph reconstruction from network data. *Communications Physics* **4**, 135 (2021).

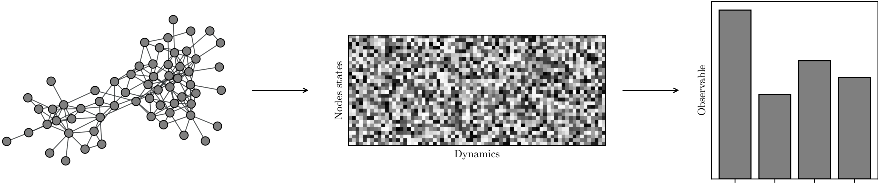
Summary

- Don't mix behaviors and mechanisms
- Emergence of synergistic signatures of higher-order mechanisms
- Higher-order signatures are invisible to low-order observables
- These signatures are a "weak" emergent phenomena
- Can be useful for downstream tasks (e.g. detection)

General picture:
Intricate relation between mechanisms and behaviors.

What can be done better?

Address the hard task

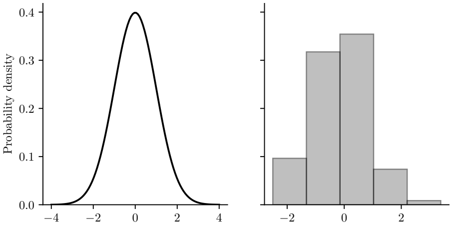


How do we actually compute the observables?

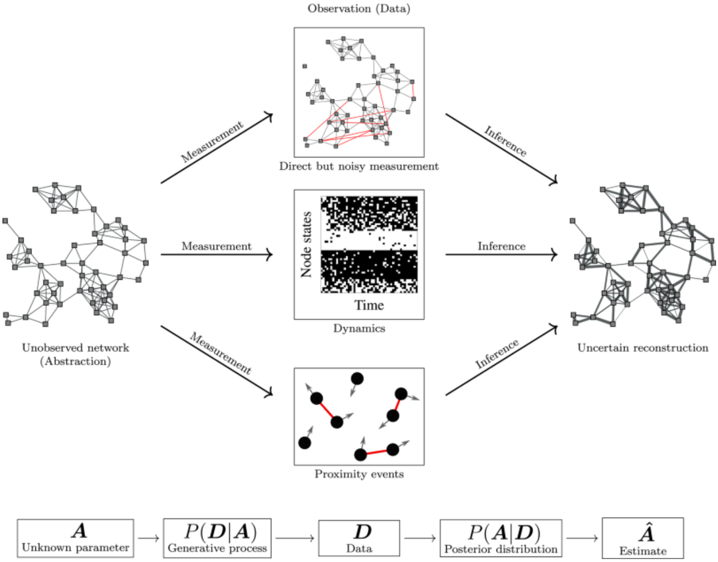
$$\mathcal{H}(X) = -\sum_{x \in \Sigma} p(x) \log p(x)$$

Given a set of M observations $\{x_i\} \in \{0,1\}^M$:

$$\mathcal{H}(X) = -\sum_{x \in \{0,1\}} p_x \log p_x \text{ where } p_x = \frac{1}{M} \sum_{i=1}^{M} \delta_{x_i,x}$$



Inverse problem[9]



[9] Peel, L. *et al.* Statistical inference links data and theory in network science. *Nature Communications* **13,** 6794 (2022).

Bayesian approach

We can only make hypotheses and evaluate to which extent the data supports them.

$$P(\mathbf{A}|\mathbf{D}) = \frac{P(\mathbf{D}|\mathbf{A})P(\mathbf{A})}{P(\mathbf{D})}$$

or if we have a model for the dynamics[10] with parameters $\{d\}$:

$$P(A, \{d\}|\mathbf{D}) = \frac{P(\mathbf{D}|\mathbf{A}, \{d\})P(\mathbf{A}, \{d\})}{P(\mathbf{D})}$$

[10] Peixoto, T. P. Network reconstruction and community detection from dynamics. *Physical review letters* **123,** 128301 (2019).

Wrapping up

- "Network" language can be *too intuitive* $\rightarrow$ fix terminology behaviors/mechanisms
- Relation between synergistic behavior and group mechanisms
- **No "model-free" analysis**

`thomas.robiglio@it-u.at`
`thomasrobiglio.github.io`
`skewed.de/lab`


Matteo Neri


Cosimo Agostinelli


Davide Coppes


Federico Battiston


Maxime Lucas


Giovanni Petri


Tiago P. Peixoto